WELCOME

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What is P/T?

Method of applying a large compression force to a structure (concrete) before loading occurs.

Applying that force AFTER the concrete has already hardened.

May also apply bending forces - to try to counteract bending created when load is applied.

BASICS OF PRESTRESSING
Basics of Prestressing

2 methods: \{ 
- pre-tensioning
- post-tensioning

2 types: \{ 
- bonded
- unbonded

Pre-tensioned:

Tendons are stretched and anchored to strong bulkheads,
then concrete is placed around the stretched tendon.

"tendon" = a group of tension strands or bars used for prestressing;
Tendons are stretched and anchored to strong bulkheads.
Long line precasting bed;

Multiple girders cast at once and separated by bulkheads;

strands

then concrete is placed around the stretched tendon.

Post-tensioned:

The concrete is first placed around embedded tubes or ducts

after the concrete is hardened, tendons are placed in the ducts, stretched to desired tension, and then anchored against the concrete at the ends.
The concrete is first placed around embedded tubes or ducts.

End anchorages for tendons

Draped P/T ducts
after the concrete is hardened tendons are placed in the ducts, stretched, and then anchored against the concrete at the ends.
Bonded:

The tendon is bonded to the concrete

pre-tensioned: concrete is placed directly around the bare exposed steel;

post-tensioned: grout is pumped into the duct after the strand is stretched.
pre-tensioned: concrete is placed directly around the bare exposed steel.
post-tensioned: grout is pumped into the duct after the tendon is stretched.

Unbonded:

The tendon is coated to prevent bonding to the concrete -

pre-tensioned: usually strand is covered with a plastic sheath;

post-tensioned: the duct is NOT grouted but the steel is protected by pumping grease or another coating into the duct.
pre-tensioned: usually strand is covered with a plastic sheath;
BRIDGES

Pretensioning and post-tensioning both used;

Pretensioning: almost always “bonded”, unless special conditions require less prestress - strands partially unbonded;

Post-tensioning: should be bonded or very specials measures taken to prevent tendon corrosion

Calculating Effects of Prestress on a Concrete Member
Internal forces in prestressed beam - no external loading:

No external load applied to beam: there cannot be any internal moment.

\[ M = 0 \]

Since there is no internal moment from loads - the resultant \( C \) in concrete must act on the same line of action as the \( T \) in the strand.

We “separate” the \( C \) and \( T \):

\( C \) is the compression created in the concrete

\( T \) is the tension applied to the tendon

Internal forces in prestressed beam - no external loading:

Axial load eccentricity in concrete

Considering only forces in the concrete:

No axial load, so \( C = T \)

Resulting eccentric compression causes an axial force and a moment: \( M = Ce \)
internal concrete stresses

the stress in concrete at any height “y” above the cgc:
(compression is +)

\[ f_y = \frac{T}{A} + \frac{(Te) y}{l} \]

Stress due prestressing alone

When loads are applied, the additional moments “M” create added stress, final stress at “y”:

\[ f_y = \frac{T}{A} + \frac{(Te) y}{l} + \frac{My}{l} \]

Total stress at any fiber “y”

Calculate the stress condition at midspan of beam:

\[ w := 0.5 \frac{k}{\text{ft}} \]

span: \( L := 22 \text{ft} \)

prestress from 3 strands:

\[ T := 60k \]
concrete area:

\[ A_c := h \cdot b \quad A_c = 240 \text{ in}^2 \]

\[ e_{gc} := \frac{h}{2} \]

\[ e = -8 \text{ in} \]

\[ I := b \cdot \frac{h^3}{12} \quad I = 1.152 \times 10^4 \text{ in}^4 \]

calculate stresses from prestressing alone:

\[ f_y := \frac{T}{A_c} + \frac{(T-c) \cdot e}{I} \]

\( f_y \) gives the stress at \( y \) from the cgc

at the beam top:

\[ f_{\text{top}} := \frac{T}{A_c} + \frac{(T-c) \cdot \left(\frac{h}{2}\right)}{I} \quad f_{\text{top}} = -0.25 \text{ ksi} \]

negative = tension stress

at beam bottom:

\[ f_{\text{bottom}} := \frac{T}{A_c} + \frac{(T-c) \cdot \left(-\frac{h}{2}\right)}{I} \quad f_{\text{bottom}} = 0.75 \text{ ksi} \]

compression
Load Balancing:

Draping strand:

Total stress at any fiber “y”

\[ f_y = \frac{T}{A} + \frac{(Te) y}{l} + \frac{M y}{l} \]

If the moment due prestress, \( (Te) \), is opposite in sign to the moment created by the loading \( (M) \) - the bending stresses cancel out and pure axial compression is left.

So, want: \( (Te) + M = 0 \)

or, strand eccentricity “e”:

\[ e = -\frac{M}{T} \]

the strand location “e” should vary opposite to the moment diagram due to applied loads.
For a typical moment diagram on a simply supported beam (above) the strand should be draped as below:

Precasters, however, cannot obtain a curved drape in pre-tensioned beams. The strand pattern in a precast beam is likely to look like:

Plotting the resulting moment diagrams:

There are critical locations where the moments do not cancel and bending stresses must be controlled!
For a typical moment diagram on a simply supported beam (above) the strand should be draped as below:

side view of beam and strand

**With post-tensioning:**

*It is easy to get the tendon in exactly the shape desired to cancel out the bending!*

*A hollow duct is placed in the beam in the desired shape. Since the tendon is not tightened yet, it doesn’t try to go into a straight line shape.*

---

**Where should strand be placed in the previous beam to have pure axial stress at midspan?**

We know the stress at "y":\[ f_y = \frac{T}{A_c} + \frac{(T \cdot c) \cdot y}{I} + \frac{M \cdot y}{I} \]

For uniform axial stress the bending parts must cancel:\[ \frac{(T \cdot c) \cdot y}{I} + \frac{M \cdot y}{I} = 0 \]

Solving for "e":\[ e = \frac{-M}{T} \]

\[ e = 6.05 \text{ in} \]
From the calculations -

we can see that the actual stresses in a member can be

**CONTROLLED**

by adjusting the tendon location “e” and the applied prestress force “T”.
Aim of Prestressing

control amount of tension in concrete

limit concrete tension to a level below cracking

keep "uncracked" cross section
moment of inertia stays = $I_{\text{gross}}$

Why use post tensioning?

• durability / crackfree
• stiffness / crackfree

allows longer spans
can possibly reduce substructures/piers
lower maintenance
elimination of joints
summary

pretension & post-tension .... both create prestress,
  -- pretension in precasting plant,
  -- post-tension at job site,

post-tensioning .... can exactly cancel out the bending effect
  from loads,

aim to prevent cracking: increased "$I" allows
  spanning longer distances,
  better durability;

Purpose of Workshop

what is post-tensioning?

PROGRAM: - best situations for using P/T

DESIGN: - special designs steps

CONSTRUCTION: - components and use
How is P/T Applied?

feed strands through duct, place anchor head and wedges

anchor casting embedded in concrete
jack & anchor head

anchor head
(bears on anchor casting)

wedges
seated in conical holes
place jack – feed strands thru over anchor head and wedges

jack pulls set of strands
jack pulls set of strands

2nd jack pushes against wedges anchoring strands against head

after strands are pulled (to desired force/elongation)

then main jack is released

extended strands are locked in position

mix grout and pump grout into duct to protect strand
alternate to strand

pull bar with jack, tighten nut in place

bearing plate applies compression to concrete

when to use

CRACKING  -- possible situation where cracking might develop and affect:

 durability, or
 performance
ideal application of post-tensioning with bars

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plan from WisDOT

target piers

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plan from WisDOT
Section BB

36 - #11 epoxy coated rebar
Cracking:

due to weight of girders alone
(deck not formed or poured)
6’-8” above footing

10’-11” above footing
cracking = reduced moment of inertia & increased deflections

pier cracking — additional softening and deflection

1.2" end deflection
Other Prime Applications

Brady Street Bridge: pedestrian pathway on Milw lakeshore

LONG SPAN
125 ft center span eliminated need for a midspan pier
5 tendons, each with nine 0.6" strands
Lakeshore State Park Bridge - near Milw lakeshore

from AECOM
MUST HAVE:

vent tubes at high point to allow air escape as grout is pumped in.
feed strand through ducts

1. strand spool
2. feeder
3. into duct

anchor head & wedges
EXAMPLE

Bridge Design & Construction Controls

Spliced – Girder Bridge

• using precast girders
• to allow long span construction
• to improve long term durability
• to simplify transport of girders
Florida - Choctawhatchee Bay

Spliced girders

Cantilevered girders

post-ten ducts

back span support

LONG SPAN POSSIBILITIES

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Florida - Choctawhatchee Bay

Spliced girders

drop-in span girder

tapered cantilever girder

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Spliced girders

I-15
Salt Lake City

over 200 ft. span

Rock Cut Bridge, Washington
Spliced girders

Rock Cut Bridge, three 63', 40 ton girders spliced for 190' span

Twist bridge, Washington
176' span, 95" girder

eliminated a center pier in stream bed
Methow River, WA, double 180’ spans, 83” girder

Nebraska Bridge: 207 ft. span, 79” I-girders
Nebraska Bridge: 207 ft. span, 79” I-girders

Spliced girders

PCI Journal Nov-Dec ’06

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Nebraska Bridge: 207 ft. span, 79” I-girders

Spliced girders

PCI Journal Nov-Dec ’06

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Post-Tensioning Bridge Workshop

splicing girders is particularly effective if:

1. it is possible to completely eliminate a middle pier by using long spans ($$$)

2. it makes shipping long span girders possible

**design & construction problem:**

Two Span Bridge
82W girders
LRFD Design

[Diagram of a two-span bridge with 180 FT ctr-ctr and 180 FT ctr-ctr dimensions]
**Design Concept:**

The 180ft length is excessive for shipping.

-- a single 180ft girder weighs 92 tons

-- a 180 ft. length is difficulty to transport around turns in a road

---

**Temporary bents - support girders**

The girders will be prestensioned to carry their own weight and the deck weight as simply supported on approx 120 ft spans.

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I-15
Salt Lake City

temporary const. supports at splice points

a single span over 200'

Alternate to temporary supports:

Step 2

Step 3

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Alternate to temporary supports:

Step 4

post-tension

Step 5

deching

Alternate to temporary supports:

suspended girder

girder hangs from steel strut beam

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Alternate to temporary supports:

good for situations where P/T is applied before deck is placed
(then deck has no prestress)

temp supports struts:
1. capacity to carry beam + deck?
2. interfere with deck placement
**Design Concept:**

After placement of the deck, the girders will be post-tensioned to form a continuous 360ft girder on two spans. Over the center pier the post-tensioning will be designed to prevent cracking in the composite bridge deck.

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**Design Concept:**

Since the deck will be part of the post-tensioned system, provision will be made for grinding of the deck and placement of a future 3in overlay, in lieu of possible deck replacement. The combination of a flexural crack free deck and overlays with age should provide a 75-100 year life.

---

**the deck is prestressed with the girders:**

we can’t easily rip it off and replace it in 20 years, make provision for an overlay in design loads (similar to method used in segmental box bridges)

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**Design Concept:**

![Diagram of bridge design](image)

42.5 FT width, 5 @ 7.5ft

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**Girder spans:**

![Diagram of girder spans](image)

L = 180 ft

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Initially each end girder is a simply supported span, the middle girder is on three supports, all precast segments are the same length

<table>
<thead>
<tr>
<th>total girder length</th>
<th>TL := 2.8in + 2L</th>
<th>incl. 2 bearings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TL = 361.333-ft</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>total length of precast girders</th>
<th>TP := TL - 2.18in</th>
<th>TP = 358.333-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>each precast girder</td>
<td>GL := ( \frac{TP}{3} )</td>
<td>GL = 119.444-ft</td>
</tr>
</tbody>
</table>
FOR DESIGN: analyze this structure with girder and deck weight

(solve for internal moments and initial locked-in stresses in girder)

girder carries all load, deck is “wet”
after the deck is cast (and diaphragms) (ctr-ctr bearings)

\[ L = 180 \text{ ft} \]

FOR DESIGN: all loads that are applied after the deck has hardened and P/T applied -
create moments in this two span continuous composite beam

Construction Sequence:

1. girders erected, on temporary supports,
   slab poured
   simply supported end spans, continuous span over pier,
   I of girder, weight of girder, slab, diaph,

2. girders and deck post tensioned,
girders become continuous over their full length,
girders lift off of temporary supports

3. additional DL, parapet + wearing surface added
   I composite girder and deck

4. live load added
   I composite girder and deck

the construction sequence directly affects the design process
Design Process:

the design process shown here follows the same steps as shown in WBM design of W72 beam in Chapter 19

Live loads

For Strength-I and Service-I:

- HL-93 loading truck + lane
- tandem + lane
- special M truck train

DLA of 33% applied to truck or tandem, but not to lane

For Fatigue:

- not necessary to check fatigue of strand in prestressed girder when tensile stress in concrete is limited

Permit Truck:

Wisconsin requires check of capacity to carry the Wis-SPV permit vehicle at 190klps

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Load distribution to girders:

AASHTO Type "k" bridge

LRFD
4.6.2.2.1

(k)

interior beam
distribution factors:

One Design Lane Loaded:

\[ 0.06 + \left( \frac{S}{L} \right)^{0.4} \left( \frac{S}{L} \right)^{0.4} \left( \frac{K_s}{12.0 L} \right)^{0.3} \]

Two or More Design Lanes Loaded:

\[ 0.075 + \left( \frac{S}{9.5} \right)^{0.6} \left( \frac{S}{L} \right)^{0.4} \left( \frac{K_s}{12.0 L} \right)^{0.3} \]

Interior beams:

one lane loaded

\[ g_{1i} = 0.06 + \left( \frac{s}{14\text{ft}} \right)^4 \left( \frac{s}{L} \right)^3 \left( \frac{K_g}{(L-t_{se})^3} \right)^{-1} \]

\[ g_{1i} = 0.404 \]

two or more lanes loaded

\[ g_{2i} = 0.075 + \left( \frac{s}{9.5\text{ft}} \right)^6 \left( \frac{s}{L} \right)^2 \left( \frac{K_g}{(L-t_{se})^3} \right)^{-1} \]

\[ g_{2i} = 0.601 \]

\[ g_i = \max(g_{1i}, g_{2i}) \]

NOTE: the distribution factors above already have a multiple lane factor included.

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exterior beams:

AASHTO gives the dist. factor "g" as:

$$ g = e 	ext{ Recorrer} $$
$$ e = 0.77 + \frac{d_e}{9.1} $$

for multiple lanes

distance from ctr of girder to barrier:

$$ d_e := s_{oh} - \frac{(w_b - w)}{2} $$
$$ d_e = 15 \text{ in} $$

$$ e := 0.77 + \frac{d_e}{9.1\text{ft}} $$
$$ e = 0.907 $$

distribution factor

$$ g_X := e \cdot g_I $$

multi-presence factor included

OR the factor from the lever rule for the one lane loaded case

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exterior lever rule

$$ s_{oh} = 30 \text{ in} $$
$$ d_e = 15 \text{ in} $$

wheel spacing = 6\text{ft}

$$ s = 90 \text{ in} $$

$$ R := 1 \left( \frac{s - s_w + s_{w2}}{2} \right) $$
$$ R = 0.5 $$

% of a lane load

add single lane multi-presence factor

$$ m = 1.2 $$

LRFD

3.5.1.1.2

$$ g_X := \max(g_X, 1.2 \cdot R) $$

Note: exterior girders and interior girders have same distribution factors, interior has larger DL and will control the design.

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### Analysis cases:

<table>
<thead>
<tr>
<th>label</th>
<th>loading:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>dead load girder and deck and diaph</td>
</tr>
<tr>
<td>B</td>
<td>dead load overlay, parapet</td>
</tr>
<tr>
<td>C</td>
<td>live load truck</td>
</tr>
<tr>
<td>D</td>
<td>live load tandem</td>
</tr>
<tr>
<td>E</td>
<td>live load special -M truck train</td>
</tr>
<tr>
<td>F</td>
<td>live load uniform lane load on span 1</td>
</tr>
<tr>
<td>G</td>
<td>live load uniform lane load on span 2</td>
</tr>
<tr>
<td>I</td>
<td>uniform unit load along span 1</td>
</tr>
<tr>
<td>J</td>
<td>uniform unit load along span 2</td>
</tr>
<tr>
<td>K</td>
<td>point load: removal of temp. bent</td>
</tr>
</tbody>
</table>

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</tr>
<tr>
<td>G</td>
<td>live load uniform lane load on span 2</td>
</tr>
<tr>
<td>I</td>
<td>uniform unit load along span 1</td>
</tr>
<tr>
<td>J</td>
<td>uniform unit load along span 2</td>
</tr>
<tr>
<td>K</td>
<td>point load: removal of temp. bent</td>
</tr>
</tbody>
</table>

---

**Case A:** Girders are not spliced and are supported on temporary bents  
**Case K:** After P/T the girder lifts off the bent, bent reaction is removed  
**Case B:** remaining DL is applied to the composite girder  
**Cases C-G:** LL is applied to composite girders  

*Note: cases F&G can be derived from I&J by multiplying the results from I&J by the uniform lane load, case B can be derived from I&J by multiplying by the overlay and parapet uniform loads.*

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after deck and joints are placed:
post-tension strands are fed through ducts

then tendons are pulled with the jack
- tendency is for strand to straighten -
post-tensioning lifts girders off temp supports!

simulate the effect of temporary support removal
by applying reverse loads
moments from removal of temporary supports (reaction gone)

Moment from Girder and Deck (on temporary supports)

temp support moment

temp support reaction on girder

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RESULTING PEAK MOMENTS (all in ft-kips):

<table>
<thead>
<tr>
<th>Lane Loading</th>
<th>Service 1 Moments per lane (ft-kips)</th>
<th>Interior Girder (ft-kips)</th>
<th>Exterior Girder (ft-kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>truck-lane</td>
<td>+5205 -4231</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tandem-lane</td>
<td>+4327 -3697</td>
<td></td>
<td></td>
</tr>
<tr>
<td>special</td>
<td>-M truck+lane (90%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2488</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Load Combinations

<table>
<thead>
<tr>
<th>Limit State</th>
<th>Case and loading type:</th>
<th>Service 1</th>
<th>Service 2</th>
<th>Service 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength 1</td>
<td>1A.2 ... A+B+C+F+K</td>
<td>+M 11043</td>
<td>-9750</td>
<td>5164</td>
</tr>
<tr>
<td></td>
<td>DL nc+DL c+truck+lane1+point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>588 ... A+B+C+F+G+K</td>
<td>-M 9910</td>
<td>-9206</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DL nc+DL c+truck+lane1+lane2+point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+M 5441</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DL c+truck+lane1+point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+M 3934</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DL c+truck+lane1+lane2+point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-M 6976</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Service 1</td>
<td>11613 ... B+C+F+K</td>
<td>+M 5760</td>
<td>3034</td>
<td>5184</td>
</tr>
<tr>
<td></td>
<td>DL c+truck+lane1+point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+M 2753</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DL on composite beam</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-M -5667</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Service 3</td>
<td>21823 ... B+C+F+K</td>
<td>+M 5164</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DL c+truck+lane1+point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+M 4847</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Service non-comp initial DL</td>
<td>31832 ... A, end girder</td>
<td>+M 3223</td>
<td>452</td>
<td>3072</td>
</tr>
<tr>
<td></td>
<td>DL nc dead load before post-tens, mid girder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+M 2077</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-M -743</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: DL=Non-composite DL, DL c=Composite DL
How to approach the design? – where to start

**Preliminary design info:**

### Controlling design criteria:

- $A_0$ at transfer, precasting plant ........T is maximum, little loading (pretension only)

| Avoid high initial tension or compression with initial concrete strength. |
| Load = $T_{initial}$ (before losses) + $M_g$ (due girder weight) |

- $M_g$ is for 133 ft girder, simply supported non-composite

---

**Controlling design criteria:**

- $R_u$ after erection with deck poured, no P/T ..... low pretension, DL

| Avoid cracking or high compression due DL with 28 day strength |
| Added load = deck weight, temp supports non-composite |

*pretension in girders only needs to be sufficient to carry girder and deck weight on the initial short temp spans!*

---

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Controlling design criteria:

C. after post-tensioning .............. T is maximum, only DL applied

DL of girder and deck

avoid high initial tension or compression with 28-day concrete strength and high prestress,
check girder and deck stress

Full DL, 2 continuous spans, composite

check this case:
not likely to control design

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Controlling design criteria:

D. at full service load, after 50 years ........ T is minimum, load is max

avoid cracking and limit concrete stress

Load = T_{final} (after losses) + M (max service moment)
composite girder

cracking in deck over pier (-M)
& comp in bottom flange at pier

this will be controlling case for P/T design

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summary ... design approach:

• design pre-tension to eliminate cracking of girders when simply supported, girder + deck weight

• design P/T to avoid tension in deck over top of pier

Preliminary design steps:

Using the prestress stress equation, AND the design criteria above:

1.) design amount of pretension to prevent tension at bottom of beam after deck pour

2.) design amount of post-tension needed to control tension of continuous beam after 75 years with full live load.

3.) develop layout for post-tensioning along the beam length

4.) calculate accurate estimate of prestress losses

5.) revise/adjust pre and post tension design to avoid premature failure at time of prestress application

6.) design eccentricity of pre-tensioned strands at end to avoid tension or compression overstress at transfer, design complete layout of post-tensioning tendons

Under Service Load conditions
Step 1) 1.) design amount of pretension to prevent tension at bottom of beam after deck pour .... BEAM IS NON-COMPOSITE near center span, after 28 days
T = remaining effective prestress, aim for little tension, Moments: Service 1 for compression, Service 3 for tension, Use interior girders for design
girder weight plus concrete deck

**non-composite girders**

BEFORE DECK IS HARDENED WEIGHT IS CARRIED BY GIRDER (NON-COMPOSITE)

**moment (kip-ft)**

**distance (ft)**

interior - exterior

**START WITH DESIGN FOR END SPAN GIRDERS** focus on the controlling interior girder (highest moments)

Calculate the stress at the bottom of the beam due to non-comp loading: (Service 3 condition)

\[
f_b = \frac{M_{ncp1}}{S_b}
\]

\[
f_b = -1.695\text{ ksi}
\]
Initial Stresses due Non-Comp. DL (without pretension)

Distance (ft)

Stress (ksi)

Girder weight plus concrete deck

High tension on bottom is unacceptable = cracking

\[ f_b = \frac{M_{ncpi}}{S_b} \]

\[ f_b = -1.695 \text{ ksi} \]

Stress at bottom due to pretensioning:

\[ f_{bp} := \frac{T}{A_g} \left( 1 + e^{-\frac{y_b}{f_{sq}}} \right) \]

and

\[ f_{bp} := f_b \]

desired final prestress

We want this to balance out the tension stress calculated above from the loading.
BUT now we are looking at time of deck pour: 40 days, and pretension loss is less (time) than for 75 years use a reduced loss

\[ \text{loss}_{\text{const}} := 6\% \]

\[ f_{\text{bpi}} := \frac{f_{\text{bp}}}{1 - \text{loss}_{\text{const}}} \quad f_{\text{bpi}} = 1.803 \cdot \text{ksi} \]

desired bottom prestress (initial) to avoid all tension

**AASHTO allows an initial tension stress, without cracks**

\[ f_{\text{tall}} := -0.19 \text{ ksi} \cdot \sqrt{\frac{f_c}{\text{ksi}}} \]

\[ f_{\text{tall}} = -0.537 \cdot \text{ksi} \]

where the 28-day strength is used for concrete just before the P/T is applied (60 days)

then the initial desired bottom compression due to the pre-tension becomes:

\[ f_{\text{bpi}} := f_{\text{bpi}} + f_{\text{tall}} \quad f_{\text{bpi}} = 1.266 \cdot \text{ksi} \]
stress for a group of strands can be found from the $f_{pp}$ equation first given above:

### 82W girder stress effects

Initial force per strand = 43.94k

<table>
<thead>
<tr>
<th>strands</th>
<th>e</th>
<th>bottom stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>-36.82</td>
<td>1.588ksi</td>
</tr>
<tr>
<td>16</td>
<td>-36.68</td>
<td>1.810ksi</td>
</tr>
<tr>
<td>18</td>
<td>-36.79</td>
<td>2.040ksi</td>
</tr>
<tr>
<td>20</td>
<td>-36.68</td>
<td>2.263ksi</td>
</tr>
<tr>
<td>22</td>
<td>-36.41</td>
<td>2.478ksi</td>
</tr>
<tr>
<td>24</td>
<td>-36.18</td>
<td>2.693ksi</td>
</tr>
</tbody>
</table>

Solution:

try 12 strands, 0.6in diameter,
initial prestress at bottom = 1.373 ksi, actual tension should be less than allowed.

$n_{se} := 12$

e_{se} := -37.68 in

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Temp: Girder and Deck Weight Initial Stresses (with pretension)

bottom tension is acceptable = no cracking
NOW LOOK AT DESIGN FOR MIDDLE SPAN GIRDER

Calculate the stress at the top of the beam due to non-comp loading over the pier (Service 3 condition)

\[
\frac{M_{ncnt}}{S_t} = 0.451 \text{ ksi}
\]

but \( f_{all} = 0.537 \text{ ksi} \)
no pretension is needed

IF - the girder is lifted at its ends:

high bending at middle would require bottom pre-tension

BUT: later on with full loading on the structure the –M over the pier will create high compression at bottom

- don't want comp. at bottom from pre-tension -
IF - the girder is lifted at its ends:

If the girder is designed to be lifted at its ends:

\[ M_g = 1.821 \times 10^7 \text{ ft.k} \]

\[ f_b = \frac{M_g}{A_b} = 0.058 \text{ ksi} \]

required pretension stress:

\[ \left| f_b - f_{ult} \right| = 0.42 \text{ ksi} \]

BUT this will later add to the total compression in the girder bottom caused by -M over the pier, and add tension in the top of the girder; both may be undesirable.

During erection the beam will be lifted, to minimize the lifting moment - select the lifting points to create two cantilevers and a middle section with equal +M and -M:

cantilever moment:

\[ M_c = -w_g \left( \frac{L^2}{2} \right) = -312.404 \text{ ft.kip} \]

middle moment:

\[ M_m = -w_g \left( \frac{L_m^2}{8} + \frac{L^2}{2} \right) = 312.404 \text{ ft.kip} \]

causing tension at the bottom at midspan and at top at lift point:

\[ f_{bb} = \frac{M_m}{A_b} = -0.164 \text{ ksi} \]

\[ f_{bt} = \frac{M_c}{A_t} = -0.175 \text{ ksi} \]
Pre-tension design

Summary:

the 2 end span girders have 12 strands for pre-tension at the bottom;

the middle span girder does not require any pre-tension;

Post-tension Design

| Step 2 | 2.) design amount of post-tension needed to control tension of continuous beam after 75 years with full live load; focus on interior girder design (controlling) |

START by finding the maximum possible tendon eccentricity at the location of maximum moment (-M) over the pier.

THEN calculate the amount of post-tension force needed with that eccentricity to counteract the tension stresses caused by the load induced moment at top of the girder and in the deck.
at the pier: \[ y_{cgh} = 34.513 \text{ in} \] to top of deck
\[ y_{cgh} = 26.013 \text{ in} \] to girder flange

Assume that 3 ducts will be used - 12 to 15 strands each, duct outer diameter = 3.4 in.

\[ N_d = 3 \quad d_d = 3.4\text{in} \]

eccentricity of strands in duct
\[ e_{sd} = .48\text{in} \]

Note:
with W82 girder web is 6.5" wide, 3.4" duct is largest to fit! limit ~ 12 strands

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duct eccentricities (keep ducts in girder):
\[ e_{d1} := y_{cgh} - 1\text{in} - \frac{d_d}{2} - e_{sd} = 22.833 \text{ in} \] top duct
\[ e_{d2} := e_{d1} - 1\text{in} - d_d = 18.433 \text{ in} \] middle duct
\[ e_{d3} := e_{d2} - 1\text{in} - d_d = 14.033 \text{ in} \] lower duct

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ducts over pier

duct eccentricities (keep ducts in girder):

\[ e_{d1} = \gamma_{cgb} - \frac{d_y}{2} - e_{ed} = 22.833 \text{ in} \]  
(top duct)

\[ e_{d2} = e_{d1} - 1\text{ in} - d_3 = 18.433 \text{ in} \]  
(middle duct)

\[ e_{d3} = e_{d2} - 1\text{ in} - d_3 = 14.033 \text{ in} \]  
(lower duct)

---

girder end - anchorages

---

total tendon eccentricity:

\[ e_{npt} := \frac{e_{d1} + e_{d2} + e_{d3}}{3} = 18.433 \text{ in} \]

from c.g. of composite

the girder top kern:

\[ k_{tcg} := \frac{-r_{sqci}}{\gamma_{cgbj}} = 19.621 \text{ in} \]

---

**note:** the "e" is less than the "kern". That means that the P/T will cause compression at the bottom of the beam, the P/E moments will also!
stress in deck due loading:

\[ f_d := \frac{M_{s3ni}}{S_{cgti}} = -2.105 \text{ ksi} \quad \text{tension} \]

post-tension compression stress needed at top:

allowable deck tension:

\[ f_{tda} = -0.48 \text{ ksi} \]

\[ f_{ptd} := - (f_d - f_{tda}) = 1.625 \text{ ksi} \]

solving for post-tensioning force needed:

\[ T_{pt} := f_{ptd} \cdot A_{cgi} \cdot \frac{1}{1 + e_{npt} \cdot \frac{y_{cgti}}{r_{sqci}}} = 1.55 \times 10^3 \cdot k \]

then the initial force needed is:

\[ T_{pto} := \frac{T_{pt}}{1 - \text{loss}_{pt}} = 1.722 \times 10^3 \cdot k \]
the initial P/T force per strand:

\[ T_{pts} := f_{tea} \cdot A_s = 43.357 \cdot k \]

the number of strands:

\[ n_{pt} := \frac{T_{pto}}{T_{pts}} = 39.728 \]

\[ n_{pt} := \text{ceil} \left( \frac{n_{pt}}{3} \right) = 14 \, \text{strands per tendon} \]

---

**Post-Tensioning**

Post-tension preliminary selection completed - based on controlling tension in the deck over the piers. Duct location over the pier determined and the required tendon size was determined.

Now check stresses in other locations at the pier to verify the acceptability of the initial design.
Check compression stress at the bottom of the girder.

check comp. stress in girder bottom fiber (Serv1) over the pier full loads at 75 years:

with the non-composite permanent DL (no pretension used):

\[ f_b = \frac{M_{ncnl}}{S_b} = 0.423 \text{ ksi} \]

due to post tension alone:

\[ f_{b,post} := \frac{T_{pt}}{A_{cgl}} + \frac{T_{pt} \epsilon_{npt}}{S_{cgb}} = 0.066 \text{ ksi} \]

stress due permanent loads (Serv1 DL) and prestress:

\[ f_b = f_b + f_{b,post} + \frac{M_{nlndl}}{S_{cgb}} = 3.078 \text{ ksi} \]

AASHTO limit on compression stress due to permanent loads and prestress:

\[ f_{cp} = 3.6 \text{ ksi} \] OK

---

**Post-Tensioning**

**Step 3** 3) develop a layout for the post-tensioning along the entire beam length;

We have the P/T form designed and the layout over the center pier where the largest bending moment occurred.

\[ \epsilon_{npt} = 18,433 \text{ in} \] above the cg of the composite beam

the tendon is composed of three separate ducts:

\[ e_{d1} = 22,833 \text{ in} \]
\[ e_{d2} = 18,433 \text{ in} \]
\[ e_{d3} = 14,033 \text{ in} \]

all measured from the composite cg

---

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Post-Tensioning

For simplicity, design a tendon profile that is concordant:
i.e. no secondary moments (or reactions at the middle pier) are created when the tendon is post-tensioned.

To find a possible concurrent tendon profile --- use the shape of the moment diagram generated by applying a unit uniform load to the entire beam (which we actually already have if load cases "I" and "J" are combined).

A tendon that was in the exact shape of the moment diagram shown is concordant.

BUT - there is a big kink over the pier
Need to eliminate the 225k reaction:

From the beam analysis with uniform load, the reaction at the pier is 225k,

apply this as a uniform upward load over an 18 ft length on either side of the pier and re-analyze for moments (along with the original unit uniform down load on the entire span).

$$w_{up} := \frac{225}{36} = 6.25 \text{ k/ft}$$
the maximum moment at the support was -3131 ft-kip, normalize this to create the desired tendon eccentricity

\[ e_{npt} = 18.433 \text{ in} \]

\[ \text{factor} := \frac{e_{npt}}{3131 \text{ ft} \cdot \text{k}} = 5.887 \times 10^{-3} \text{ in/ft} \cdot \text{k} \]

that creates an upward loop of 18.43 in. at pier a downward max sag of -13.2 in. at 67 ft, no sag at 134 ft, a positive rise of 13.4 in. at 162 ft where the second curve starts, and a slope of 0.541 in./ft at 162 ft.
With the tendon layout selected, we can calculate the top and bottom stress at every point along the span and compare with the allowed stresses:

For checking tension at top or bottom ....

1) calculate the stress due to non-composite M acting on the girder alone, (DL+pretension)
2) calculate stress at the same location due to the composite M acting on the composite girder (added DL, LL and post-ten)
3) combine the stresses above to get total stress

---

**Step 4**  4.) *Calculate an accurate estimate of prestress losses*

**Prestress loss calculation:**

1. Friction and anchorage set loss;
2. Elastic Shortening (ES), shortening of the beam as soon as prestress is applied; can this be compensated for by overstressing?
3. Shrinkage (SH), shortening of concrete as it hardens, time function;
4. Creep (CR), slow shortening of concrete due to permanent compression stresses in the beam, time function;
5. Relaxation (RE), the tendon slowly accommodates itself to the stretch and the internal stress drops with time;
Calculate the prestress loss step-by-step. First look at the loss in the post-tensioning due to anchor set and friction along the tendon. See if those losses can be partially compensated for by overjacking from one end or by slightly overjacking from both ends.

Then estimate the elastic shortening loss during post-tensioning. Can this loss also be offset by overjacking some of the strands to begin with?

Knowing the resulting forces from the post-tensioning, the total elastic losses in the pre-tensioned strands can be calculated.

Finally, the shrinkage, creep, and relaxation time dependent losses will be calculated.

1. Friction loss

From the tendon profile information, which was based on a moment diagram, we know the tendon slope and location at various points along the span.

All friction calcs will be based on the "tendon" profile, not the separate duct profiles.

At the left end: tendon slope = -0.0328 rad
at the inflection point the slope = +0.0466 rad (at 162ft.)
over the pier the slope = 0 rad
Calculate the angle change:

in segment 1, from \( x = 0 \) to 162 ft:

\[
\alpha_1 := 0.0466 - (-0.0328) = 0.079 \text{ radians}
\]

\[
L_1 := 162 \text{ ft}
\]

in segment 2, from \( x = 162 \) ft to pier:

\[
\alpha_2 := 0.0466 \text{ radians}
\]

\[
L_2 := 18 \text{ ft}
\]

Friction coef and wobble coef, using corrugated semirigid sheathing:

\[
k_W := 0.0002 \frac{1}{\text{ft}} \text{ wobble}
\]

\[
\mu := 0.20 \text{ friction coef}
\]

the % of prestress remaining after the first curve is:

\[
FR_1 := \exp[-(k_W \cdot L_1 + \mu \cdot \alpha_1)] = 95.287\% 
\]

the % of prestress remaining after the second curve is:

\[
FR_2 := \exp[-(k_W \cdot L_2 + \mu \cdot \alpha_2)] = 98.716\% 
\]
the % of prestress remaining after the first curve is:
\[ FR_1 = \exp[-(K_w L_1 + \mu \alpha_1)] = 95.297\% \]
the % of prestress remaining after the second curve is:
\[ FR_2 = \exp[-(K_w L_2 + \mu \alpha_2)] = 98.716\% \]

so the % remaining at the center pier is:
\[ FR := FR_2 \cdot FR_1 = 94.064\% \]
\[ \text{loss}_{FR} := 1 - FR = 5.936\% \]

Which is very close to our original estimate.

original friction loss estimate was: \[ \text{loss}_{FR} = 5.446\% \]

To compensate for the friction loss, the tendons may be overstressed when initially jacked and then slightly released before anchoring.

The AASHTO stressing limits are:
short term prior to seating ...
\[ f_{\text{tia}} := 0.90 \cdot f_{py} = 218.7 \text{ ksi} \]
at anchorage just after seating ..
\[ 0.70 \cdot f_{pu} = 189 \text{ ksi} \]
elsewhere just after seating ...
\[ f_{\text{tea}} := 0.74 \cdot f_{pu} = 199.8 \text{ ksi} \]
If strand is initially jacked to the limit, the stress at the center pier would be:

\[ f_{\text{pier}} := FR \cdot f_{\text{tia}} = 205.717 \text{ ksi} \]

but we want to limit this to \( f_{\text{tea}} \), so jacking stress would be:

\[ f_{\text{jack}} := \frac{f_{\text{tea}}}{f_{\text{pier}}} f_{\text{tia}} = 212.41 \text{ ksi} \]

the stress at the opposite end of the beam would be:

\[ f_{\text{pier}} := FR \cdot f_{\text{jack}} = 199.8 \text{ ksi} \quad \text{OK} \]

\[ f_{\text{end}} := f_{\text{jack}} \cdot \left( FR_1 \cdot FR_2 \cdot FR_2 \cdot FR_1 \right) = 187.939 \text{ ksi} \]

which is less than \( f_{\text{tea}} = 189 \text{ ksi} \)

\[ \text{OK} \]
We are going to over stress the tendon during jacking, so that the stress at the pier (considering loss to that point) is exactly what we want. The pier is the location of the max moment and -M that causes deck cracking. Then - we will release some of that over stress, just enough to get the stress at the anchorage back down to the allowable, but the friction will prevent stress from dropping back down at the pier ... so it will stay high where we want it to be.

We will actually end up with the highest prestress near center pier, there will be less at the far end because of additional friction loss, and also less at the jacking end because some will be released before seating the anchorage.
**Friction loss**

SO ... friction loss at center pier can be taken as 0, the stress will be equal to the allowed stress of $f_{\text{taa}}$.

Going back and using the friction loss at the pier as zero - and redesigning the post-tensioning --

we only need 13 strands per duct

and the amount of stretch in the tendon during pulling can be found by using the average stress or summing the stretch along the length:

\[
\sigma_{\text{avg}} := \frac{(f_{\text{jack}} + f_{\text{end}})}{2} = 200.174 \text{ ksi}
\]

\[
\varepsilon_{\text{avg}} := \frac{f_{\text{avg}}}{E_S} = 7.024 \times 10^{-3} \text{ in/in}
\]

**total stretch:**

\[
\Delta_{\text{stretch}} := 2 \cdot L \cdot \varepsilon_{\text{avg}} = 30.342 \text{ in}
\]

or summing the strain along the entire length with the variation of tendon stress (accurate method): 30.307 in
or summing the strain along the entire length with the variation of tendon stress (accurate method): 30.307 in

the total jacking force per tendon:

\[ T_{\text{jack}} := n_{pt} \cdot A_s \cdot f_{\text{jack}} = 599.208 \text{ kip} \]

During tendon stressing the contractor would pull the tendons to the specified force (from pressure dial gage) AND check the total elongation as given above.

---

**Anchorage set loss:**

Anchorage seating loss depends on the post-tensioning system, but will be estimated here as a 0.25 inch loss as the wedges are seated in the chucks and the strand is anchored.

\[ \text{seat} := 0.25 \text{in} \]
the downward slope of the stress line, before anchor set, can be found from the difference in stress between the jacking end and the segment end at 162 ft. ---

\[ f_{162} = FR_1 \cdot f_{\text{jack}} = 202.398 \text{ ksi} \]

\[ \text{slope} = \frac{-f_{\text{jack}} - f_{162}}{L_1} = -0.062 \frac{\text{ksi}}{\text{ft}} \]

the drop in stress at the jacking end, upon seating, is such that the shaded area in the stress diagram is equal to the product of the seating distance and strand modulus:

see references:
- Naaman - Prestressed Concrete Analysis and Design,
- OR PCI Bridge Design Manual
the drop in stress at the jacking end, upon seating, is such that the \( \Delta f_{\text{end}} \) stress is equal to the product of the seating distance and strand modulus:

\[
\Delta f_{\text{end}} \left( \frac{X}{2} \right) = E_s \cdot \text{seat}
\]

from the slope of the stress line - which is purely related to friction and will be of the opposite sign on release when the strand tries to slip back into the girder - the change in end stress will equal:

\[
\Delta f_{\text{end}} := 2 \cdot (\text{-slope}) \cdot X
\]

solving these two equations simultaneously, and hoping the \( X \) falls within the first tendon segment:

\[
\Delta f_{\text{end}} := 2 \cdot \sqrt{E_s \cdot \text{seat} \cdot (\text{-slope})} = 12.115 \text{ ksi}
\]

and:

\[
X := \frac{\Delta f_{\text{end}}}{2 \cdot (\text{-slope})} = 98.019 \text{ ft}
\]

OK - is in first segment
Tendon Stress
(with friction and seating loss)

The resulting stress at the jack end after release would be:

\[
f_{\text{jack}} = f_{\text{jack}} - \Delta f_{\text{end}} = 200.295 \text{ ksi}
\]

but the stress limit here is:

\[f_{\text{taa}} = 189 \text{ ksi}\]

NOT OK

the stress at the X will be the highest:

\[
f_X = f_{\text{jack}} + \text{slope} \cdot X = 208.352 \text{ ksi}
\]

the stress limit here is:

\[f_{\text{tea}} = 199.8 \text{ ksi}\]

NOT OK
Since the stress at the anchorage and at "X" are both too high - the jack must release some tension before the strand is anchored. The jack will still pull the strands to the same initial jacking stress to obtain the desired stress at the middle pier, but then the strands will be released slightly to drop the stress at the anchorage and other locations down to the acceptable level.

With a larger release of stress at the end, the distance "X" will extend into the mid segment, in fact we want X to be equal to the distance to the middle pier: 180ft.
then the stress at the jacking end, after seating, should be

\[ f_{\text{jackr}} := f_{\text{pier}} + \text{slope}_{\text{avg}} \cdot L = 187.19 \text{ ksi} \]

\[ f_{\text{taa}} = 189 \text{ ksi} \]

OK

total change:

\[ f_{\text{drop}} := f_{\text{jack}} - f_{\text{jackr}} = 25.219 \text{ ksi} \]

before seating it would be:

\[ f_{\text{jacks}} := f_{\text{jackr}} + \Delta f_{\text{end}} = 199.305 \text{ ksi} \]
SUMMARY: of prestress application process, jacking method.

Each strand is jacked to a force of:

\[ f_{jack} = 212.41 \text{ ksi} \]
\[ T_{jack} = 599.208 \text{ k} \]

The elongation should be:

\[ \Delta_{stretch} = 30.342 \text{ in} \]

The jacking stress should be reduced to

\[ f_{jacks} = 199.305 \text{ ksi} \]

With a force:

\[ n_{pt} A_s f_{jacks} = 562.24 \text{ k} \]

The wedges should be placed and seated, after seating, remaining elongation should be:

\[ \Delta_{stretch} - \delta_{drop} = 29.822 \text{ in} \]

And the remaining force is:

\[ n_{pt} A_s f_{jacks} = 528.084 \text{ k} \]

Elevation at beam end:

\[ h_t := h_t + t_s = 90.5 \text{ in} \]

\[ L_{gz} > h_t \]

\[ L_{gz} < 1.5 h_t \]

Near the ends of the girders the webs will be thickened to allow required bearing area for the post-tensioning anchorages. The web thickness will be made equal to the bottom flange width (30 in.). This thickness should be provided for a length at least equal to the total girder depth.
Girder anchorage zone reinforcement design:

Design reinforcing required to allow the prestress force from the post tensioning anchorages to spread across the composite girder cross section.

Use strut and tie modelling to select the required reinforcing for the disturbed stress region in the general zone near the girder end.  

First: determine the boundary forces on a section of the girder near its end due to the post-tension forces .... including force applied at the anchorages and resulting internal forces from assumed stress distribution within the girder. The boundary is taken outside "general zone".

Second: layout a strut-tie truss system inside the disturbed region and solve for the tension forces in the ties and compression in struts.
\[ T_{pt0} := A_{pt} \cdot f_{jack} = 1.809 \times 10^3 \cdot k \]

design for 1.2x this force:

\[ T_u := 1.2 \cdot T_{pt0} = 2.171 \times 10^3 \cdot k \]

AASHTO 3.4.3.2

The internal stress:

\[ f_{int} := \frac{T_u}{A_{cgi}} = 1.442 \text{ ksi} \]

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STRUTS:

Add ties:
Add secondary members – triangulation:

Co-ords for computer model:

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Post-Tensioning Bridge Workshop

EXAMPLE

Bridge Design & Construction Controls

OTHER DESIGNS
Multi Span Slab Bridges:

longitudinal reinforcing is replaced with post-tensioning

ducts
with P/T: length of middle span can be extended as long as 80-90ft.
piers can be moved from in water - to stream sides

Same design process is used:
1. find maximum eccentricity for tendon at critical section
2. solve for prestress force needed at that section
3. find a concordant tendon layout using a M diagram shape
Simpler than previous example:

- no need for temporary supports;
- no pretensioning combined with P/T;
- no variation between non-composite and composite sections;